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DESCRIPTION OF THE CREEP AND FAILURE OF BEAMS IN BENDING AND SHAFTS IN TORSION BY EQUATIONS WITH A SCALAR DAMAGE PARAMETER

B. V. Gorev and I. D. Klopotov

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Results of the theoretical and experimental investigations of beam bending by a constant moment and torsion of continuous shafts with a constant rate of the torsion angle are given. It is shown that the kinetic equations of creep and damage with a scalar parameter can be applied to the description of deformation in the presence of nonuniform stresses.

Gorev and Klopotov [1] specified the determining equations of creep with a scalar damage parameter; this makes it possible to determine this parameter from experimental tension-compression and torsion data. It was established that the processes of damage accumulation in tension and in compression are different for materials with different creep resistances.

In the present paper, we show the applicability of the equations of creep and damage [1] to the description of deformation at nonuniform stresses with the use of calculations and comparison with experimental data on pure bending of rectangular beams and torsion of continuous shafts from an AK4-1T alloy.

1. Bending of Beams by a Constant Moment. We consider pure bending of a rectangular beam of width b and height h under the action of a constant external moment M. Assuming that at an arbitrary point of the beam, the total strain is composed of elastic and creep strains at any moment of time and using the hypothesis of plane sections, from the equations of equilibrium

$$b \int_{-h/2}^{h/2} \sigma z \, dz = M, \qquad \int_{-h/2}^{h/2} \sigma \, dz = 0$$

we find the curvature of the beam æ, the neutral-line shift δ, and the stress σ acting at the point located at the distance z from the middle surface:

$$æ = \frac{M}{EJ} + \frac{b}{J} \int_{-h/2}^{h/2} \varepsilon^c z \, dz; \tag{1}$$

$$\delta = -\frac{1}{æh} \int_{-h/2}^{h/2} \varepsilon^c \, dz; \tag{2}$$

$$\sigma = Eæ(z - \delta) - E\varepsilon^c, \tag{3}$$

where E is Young's modulus of the material and $J = bh^3/12$ is the centroidal moment of inertia of the beam cross section.

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TABLE 1

Tension					Compression				
m	$B_A \cdot 10^9$, MJ/(m^3· sec)	$B_\omega \cdot 10^9$, sec^{-1}	$\zeta \cdot 10^4$, MPa^{-2}	$\beta \cdot 10^4$, MPa^{-2}	$\bar{m}$	$\bar{B}_A \cdot 10^9$, MJ/(m^3· sec)	$\bar{B}_\omega \cdot 10^9$, sec^{-1}	$\bar{\zeta} \cdot 10^4$, MPa^{-2}	$\bar{\beta} \cdot 10^4$, MPa^{-2}
2	3.0	2.014	2.09	1.611	5	8.608	0.845 75	1.515	1.2906

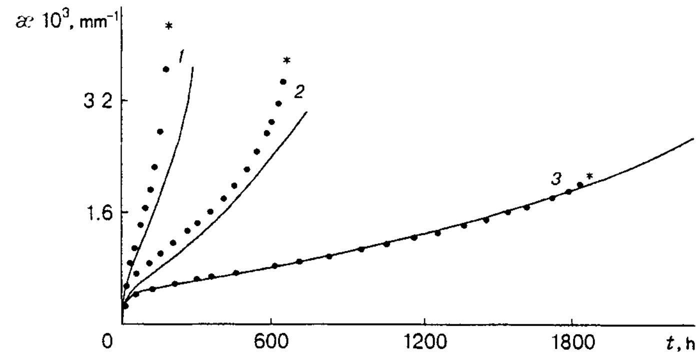

Fig. 1

Substituting the stress expression (3) into the equations of creep and damage for pure bending of the beam

$$\frac{dA}{dt} = \frac{B_A(\exp\zeta\sigma^2 - 1)}{(1-\omega)^m}\vartheta(\sigma) + \frac{\bar{B}_A(\exp\bar{\zeta}\sigma^2 - 1)}{(1-\omega)^{\bar{m}}}\vartheta(-\sigma),$$
$$\frac{d\omega}{dt} = \frac{B_\omega(\exp\beta\sigma^2 - 1)}{(1-\omega)^m}\vartheta(\sigma) + \frac{\bar{B}_\omega(\exp\bar{\beta}\sigma^2 - 1)}{(1-\omega)^{\bar{m}}}\vartheta(-\sigma), \tag{4}$$

we obtain a system of three integrodifferential equations for A, ω, and ε^c. In Eqs. (4), we have $dA/dt = (d\varepsilon^c/dt)\,\sigma$, $\vartheta(x) = 1$ for $x > 0$, and $\vartheta(x) = 0$ for $x \leqslant 0$; B_A, ζ, m, B_ω, and β and $\bar{B}_A$, $\bar{\zeta}$, $\bar{m}$, $\bar{B}_\omega$, and $\bar{\beta}$ are the tension and compression characteristics, respectively.

As done in [2], we divide the beam cross section over its height and approximate the integrals by finite sums to obtain a system of first-order ordinary differential equations, which was solved by the Runge-Kutta method. The calculations were performed for the AK4-1T characteristics given in Table 1 at 200°C [1]. The Young's modulus was $E = 60$ GPa, the beam dimensions were $200 \times 10 \times 20$ mm, and the number of divisions through the beam was $k = 64$. The initial data for $t = 0$ were $\varepsilon_k^c = A_k = \omega_k \equiv 0$.

The solid curves and points in Fig. 1 refer to the theoretical and experimental values of $\ae = \ae(t)$, respectively, for three values of the bending moment. The maximum stresses at zero time are $\sigma_{\max}(0) = 264.87$, 235.44, and 196.20 MPa (curves 1-3, respectively). The bending tests were carried out until the failure occurred (the values of $\ae_*$ corresponding to the failure time t_* are asterisked). For the measurement base $l_0 = 100$ mm, the beam curvature was calculated by the formula $\ae = 8W/l_0^2$ with the use of the experimentally measured deflection W.

Figure 2a shows diagrams of the stresses in the beam cross section which were calculated for the moments of time $t = 0$, 61, 181, and 601 h (curves 1–4, respectively) in the experiment with $\sigma_{\max}(0) = 235.44$ MPa. One can see that the stresses are redistributed over the cross section with time and at the moment before failure, the stretched fibers of the beam stop resisting the deformation. The damage distribution over the beam height is shown in Fig. 2b at various moments of time. As can be seen, the damages in the stretched region of the beam are accumulated more intensively, which leads to failure of the beam in this region.

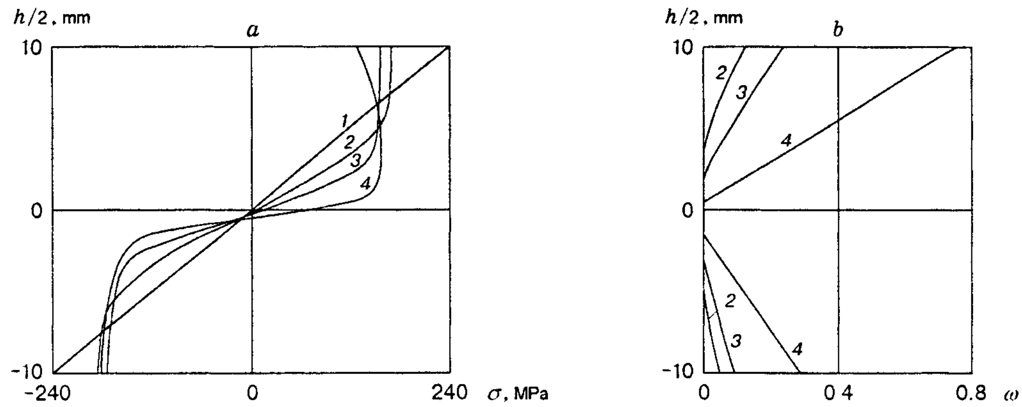

Fig. 2

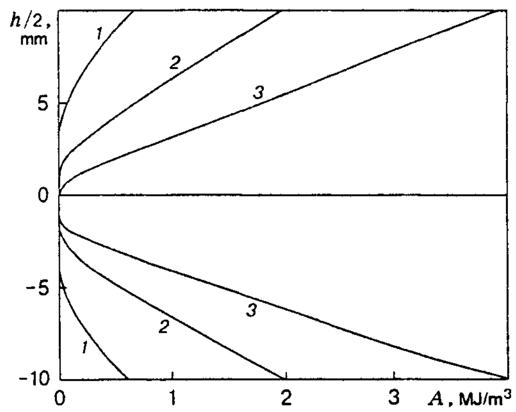

Fig. 3

Figure 3 shows the distribution of the work of dissipation in creep over the cross section in the experiment with $\sigma_{\max}(0) = 264.87$ MPa at the moments $t = 20.5$, 200, and 240 h (curves 1–3). It should be noted that the work of dissipation in the stretched and compressed fibers (located symmetrically relative to the middle surface) is almost the same, whereas the degree of damage of these fibers differs significantly.

It was established experimentally and supported by calculations that, for materials with different tensile and compression properties, the failure during pure bending begins in the border fiber subjected to tension.

2. Torsion of Shafts with a Constant Rate of the Torsion Angle. In studying the torsion of a continuous shaft, we assume that the cross sections remain plane and the radial fibers remain straight. Then, we have only the nonzero shear strain in the cylindrical coordinates, $\gamma = 2\varepsilon_{\varphi z} = \theta r$. We specify the rate of the torsion angle $\dot{\theta}$ per unit length of the shaft such that the stresses along the radius r do not exceed the elastic limit at any moment of time, i.e., as for the beam, we assume that the strain is composed of the elastic and creep components

$$\gamma = \tau/G + \gamma^c, \tag{5}$$

where G is the shear modulus. Differentiating (5) with respect to time, we obtain the following equation for calculating the shear-stress rate τ over the shaft radius depending on the shear-strain rate of creep and the specified rate of the torsion angle $\dot{\gamma}^c$

$$\frac{d\tau}{dt} = G\,(\dot{\theta} r - \dot{\gamma}^c). \tag{6}$$

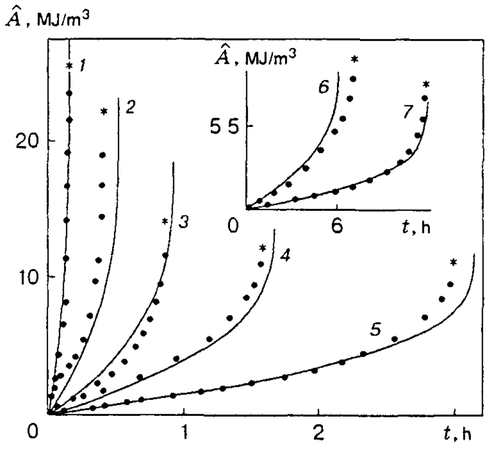

Fig. 4

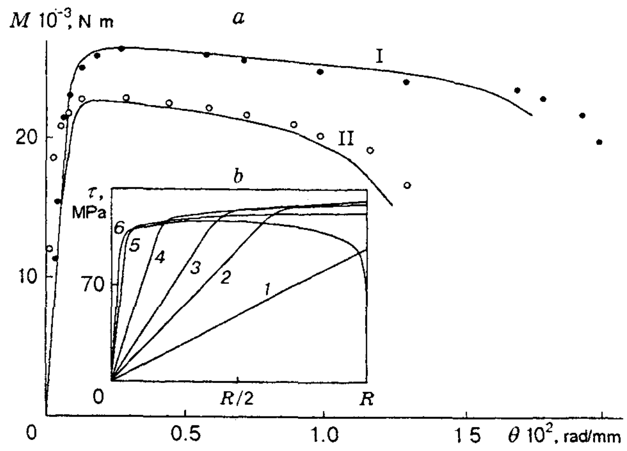

Fig. 5

The shear-strain rate is determined by the kinetic equations of creep and damage with a scalar parameter, and the coefficients of the equations for pure torsion were determined by the procedure outlined in [1]. The experiments were carried out on continuous round specimens of diameter 20 mm and working length 40 mm which were subjected to constant torque. To determine the true failure strains, we used continuous specimens, since in the tests on thin-walled specimens, at high levels of stresses, when the shear strains attain 10% and greater, one can observe the geometrical buckling of the thin-walled samples before failure.

The experimental data for continuous specimens were processed by the method of characteristic parameters [3, 4]. In torsion of a continuous specimen, the stresses redistributed. However, there exists a sufficiently small region where the material behavior can be described by the equations of uniaxial stress state $\tau - \gamma$ [3, 4]. This region is the vicinity of a point with the coordinate at the characteristic point of intersection of the elastic- and "steady"-stress diagrams

$$\hat{r} = (J_{1p}/J_{np})^{n/(n-1)}. \quad (7)$$

Here $J_{np} = 2\pi \int\limits_0^R \rho^{2+1/n} d\rho$ is the generalized polar moment of inertia of the cross section, n is the creep index, and R is the radius of the specimen.

In practice, the stress at the characteristic point does not change under stationary loading conditions and remains equal to the initial stress, i.e., to the elastic value up to failure. The calculations and experiments show that the coordinate $\hat{r}$ is almost independent of the value of the external torque and the material properties [3–6].

In accordance with the theoretically and experimentally substantiated method, the curves of uniaxial deformation (diagrams) are plotted for the values of the shear stresses and shear strains at the characteristic point $\hat{\tau} = (M/W_{1p})(\hat{r}/R)$ and $\hat{\gamma} = \theta\hat{r}$, where W_{1p} is the torsional elastic moment of resistance.

When the creep index is unknown (as in the case considered), the point of intersection of the diagrams for elastic and ideally plastic states ($n \to \infty$) can be used with sufficient accuracy [5, 6] as the characteristic point. In this case, we have $\hat{r} = 3R/4$ and the expressions for $\hat{\gamma}$ and $\hat{\tau}$ and, hence, for $\hat{\varepsilon}_i$ and $\hat{\sigma}_i$, take the following form: $\hat{\gamma} = 3\theta R/4$, $\hat{\tau} = 3M/(4W_{1p})$, $\hat{\varepsilon}_i = \sqrt{3}\theta R/4$, and $\hat{\sigma}_i = \sqrt{3}\hat{\tau}$.

The points in Fig. 4 show experimental values of the work of dissipation versus the time $\hat{A}(t)$ at the characteristic point (the asterisks refer to the moment of failure) for torsion of continuous round specimens from AK4-1T at 250°C which were subjected to the constant moment. Curves 1–7 refer to the following stress intensities at the characteristic point: $\hat{\sigma}_i$ = 230, 210, 200, 190, 180, 170, and 160 MPa. The solid curves show the approximation of the deformation curves by a power law:

$$\frac{dA}{dt} = \frac{B_A \sigma_i^n}{(1-\omega)^m}, \qquad \frac{d\omega}{dt} = \frac{B_\omega \sigma_i^k}{(1-\omega)^m}, \tag{8}$$

where $B_A = 2.79 \cdot 10^{-40}$ $\text{MPa}^{1-n} \cdot \text{sec}^{-1}$, $n = 16$, $m = 2$, $B_\omega = 3.39 \cdot 10^{-31}$ $\text{MPa}^{-k} \cdot \text{sec}^{-1}$, $k = 11.5$, and $G = 17$ GPa.

The determining equations (8) can be combined with (6) to give the system of differential equations for solving the formulated problem:

$$\frac{d\tau}{dt} = G\Big(\dot{\theta} r - \frac{(\sqrt{3})^n B_A \tau^{n-1}}{(1-\omega)^m}\Big), \qquad \frac{d\omega}{dt} = \frac{(\sqrt{3})^k B_\omega \tau^k}{(1-\omega)^m}.$$

As for the beam, calculations were performed by the Runge —Kutta method with a variable integration step; the number of the points of division from the axis along the shaft radius was equal to 32. As the initial conditions for $t = 0$, we set $\tau = 0$ and $\omega = 0$. Calculations were performed for continuous shafts having the above-mentioned dimensions for two constant rates of the torsion angle.

The points in Fig. 5a refer to experimental data for $M = M(\theta)$, and the curves to theoretical relations for the rate of the torsion angle $\dot{\theta} = 1.14 \cdot 10^{-5}$ and $1.08 \cdot 10^{-6}$ rad/(mm · sec) (curves I and II, respectively).

Figure 5b shows diagrams of the shear stresses along the radius of the shaft at the moments of time $t = 50$, 10^2, $1.5 \cdot 10^2$, $3 \cdot 10^2$, $9 \cdot 10^2$, and $1.5 \cdot 10^3$ sec (curves 1–6) for $\dot{\theta} = 1.08 \cdot 10^{-6}$ rad/(mm · sec).

The satisfactory agreement between the calculated and experimental values for the stationary and nonstationary regimes of loading in tests on beam bending and shaft torsion up to failure shows the applicability of the equations of creep and damage with the same index of loss of strength m in both equations to the description of deformation in the presence of nonuniform stresses.

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